HEAT TRANSFER IN THE HEAT TREATMENT OF CONCRETE

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Thermal-engineering calculation of the heating chamber at a polygon by numerical solution of the heat-conduction equations modeling the accelerated production of concrete components is proposed.

According to [1], cast concrete mixture attains operational properties (100% strength) in normal conditions after 28 days. A basic approach to increasing the rate of concrete production is to modify the process employed, for example, by improving the heat treatment. As is known [1], heat treatment occupies 70-80% of the production cycle. Depending on the heating conditions, the concrete may reach 70-80% of the operational strength in the course of this cycle.

To accelerate the hardening of concrete (ferroconcrete) in production conditions, convective [2] and conductive [3] heating is used. In the first case, the concrete mixture is heated in a chamber by steam–air mixture; in the second, the ferroconcrete structures are placed in the alternating electromagnetic field of the shell created by industrial-frequency current. In [2, 3], the advantages and disadvantages of both approaches were considered in an extensive review of the literature data. In particular, analytical solutions of the heat-conduction equation for concrete but with specially specified heat sources (sinks) in the energy-conservation equation were obtained. The analytical solutions obtained in [2] cannot be used to find the temperature of concrete with a thickness $H_1 > 0.3$ m or in nontraditional conditions of concrete-mixture heating, for example, outside the plant or in winter conditions of chamber operation. The aim of the present work is to determine the temperature of the concrete in nontraditional heating conditions, to simplify the three-dimensional formulation of the problem on the basis of the averaging method [4, 5], and to create an engineering method for its solution.

Formulation and Solution of the Problem. The hardening of concrete in the first approach to accelerated production is due to exothermal hydration of the cement on account of bound water [3]. The heat liberation of this reaction depends exponentially on the concrete temperature [2]. In addition, freshly cast concrete is a wet body; therefore, in the general case there may be evaporation (of free water) from the concrete.

For the sake of simplicity, it is assumed that: 1) the thermal conductivity of concrete is constant in plate heating; 2) the physical properties of the body are the same in all directions; 3) the water evaporation from the plate is uniform over the whole volume.

The mathematical problem of the heat transfer of a heated gas flux with a reactive body reduces to the solution of the system of equations

$$\rho_1 c_{p1} \frac{\partial T_1}{\partial t} = \lambda_1 \left(\frac{\partial^2 T_1}{\partial x^2} + \frac{\partial^2 T_1}{\partial y^2} + \frac{\partial^2 T_1}{\partial z^2} \right) + q_{ex} - q_{va}, \tag{1}$$

$$c_{p3}\rho_3 \frac{\partial T_3}{\partial t} = \lambda_3 \frac{\partial^2 T_3}{\partial z^2}, \tag{2}$$

$$q_{ex} = Q_{ex} \text{Ц}/t, \quad q_{va} = Q_{va}\omega, \quad [Q] = \text{kJ/kg}, \quad [\omega] = \text{kg/(m}^3\cdot\text{sec)},$$
$$Q_{ex} = 7{,}75\, Q_{28} (G/\text{Ц})^{0,44} [1 - g \exp(-f\theta)], \tag{3}$$

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$$\theta = (T_1 - 273)\tau, \quad [\theta] = \text{deg-hr}, \qquad [\tau] = \text{hr}, \quad T_1 \leqslant 363\,\text{K},$$

$$0 \leqslant \theta \leqslant 375, \; g = 1, \; f = 1{,}5 \cdot 10^{-3}; \; 375 < \theta < 2000, \; g = 0{,}666, \; f = 4 \cdot 10^{-4}.$$

If the length and width of the panel is more than 4-6 times greater than its thickness, Eq. (1) is replaced by the equation for the heating of a one-dimensional body on a steel substrate

$$\rho_i c_{pi} \frac{\partial T_i}{\partial t} = \lambda_i \frac{\partial^2 T_i}{\partial z^2} + \delta_i (q_{ex} - q_{va}), \; \delta_1 = 1, \; \delta_2 = 0, \; i = 1, \; 2. \tag{4}$$

In the opposite case, it is expedient to average over the coordinates x and y.

The following initial and boundary conditions must be taken into account in solving Eqs. (1), (2), and (4)

$$T_i|_{t=0} = T_{i\,\text{ini}} \quad i = 1, \; 2, \; 3; \tag{5}$$

1) the temperature field in the body is uniform with respect to z

$$\lambda_1 \frac{\partial T_1}{\partial z}\bigg|_{z=H_1-0} = \lambda_2 \frac{\partial T_2}{\partial z}\bigg|_{z=H_1+0}, \quad T_1|_{z=H_1-0} = T_2|_{z=H_1+0}, \tag{6}$$

$$\alpha_1 (T - T_1|_{z=0}) = -\lambda_1 \frac{\partial T_1}{\partial z}\bigg|_{z=0}, \tag{7}$$

$$\alpha_2 (T_s - T_2|_{z=H_1+H_2}) = -\lambda_2 \frac{\partial T_2}{\partial z}\bigg|_{z=H_1+H_2}, \tag{8}$$

$$\alpha_1 (T - T_3|_{z=0}) = -\lambda_3 \frac{\partial T_3}{\partial z}\bigg|_{z=0}, \tag{9}$$

$$\alpha_3 (T_0 - T_3|_{z=H_3}) = -\lambda_3 \frac{\partial T_3}{\partial z}\bigg|_{z=H_3}; \tag{10}$$

2) the temperature field in the concrete is three-dimensional, and the cartesian axes run along the edges of the block

$$\alpha_1 (T - T_1|_{F_1}) = -\lambda_1 \frac{\partial T_1}{\partial n}\bigg|_{F_1}, \tag{11}$$

$$\frac{\partial T_1}{\partial z}\bigg|_{F_2} = 0; \tag{12}$$

3) suppose that in the latter case it passes through the center of the panel with respect to x and y; then

$$\frac{\partial T_1}{\partial x}\bigg|_{x=0} = \frac{\partial T_1}{\partial y}\bigg|_{y=0} = 0, \tag{13}$$

$$\alpha_1 (T - T_1|_{x=a}) = -\lambda_1 \frac{\partial T_1}{\partial x}\bigg|_{x=a}, \tag{14}$$

$$\alpha_1 (T - T_1|_{y=b}) = -\lambda_1 \frac{\partial T_1}{\partial y}\bigg|_{y=b}, \tag{15}$$

TABLE 1. Heat-Transfer Coefficient as a Function of the Temperature of Saturated Steam–Air Mixture

$\alpha_1/1{,}163$, W/(m²·K)	0	40	80	120	200	230	300	400	500
$T-273$, K	0–27	40	55	63	73	77	83	87	91

$$\alpha_1 (T - T_1|_g) = \lambda_1 \frac{\partial T_1}{\partial z}\bigg|_g, \tag{16}$$

$$\frac{\partial T_1}{\partial z}\bigg|_{F_2} = 0. \tag{17}$$

If the averaging method of [4, 5] is used, solving the boundary problem in Eqs. (1) and (13)-(17) reduces to solving the one-dimensional heat-conduction equation with boundary conditions

$$\alpha_1 (T - T_*|_{z=0}) = -\lambda_1 \frac{\partial T_*}{\partial z}\bigg|_{z=0}, \tag{18}$$

$$\frac{\partial T_*}{\partial z}\bigg|_{z=H_1} = 0. \tag{19}$$

To this end, a trial solution

$$T_1 = T_*(z,\ t)(1 + Ax^2)(1 + Cy^2 \exp|y|), \tag{20}$$

which is known to satisfy the symmetry condition in Eq. (13), is introduced. Substituting Eq. (20) into Eq. (1) and integrating with respect to x from 0 to a and with respect to y from 0 to b, it is found that

$$\frac{\partial T_*}{\partial t} = \varkappa \frac{\partial^2 T_*}{\partial z^2} + q + T_* \varkappa \left\{ \frac{2A}{1 + Aa^2/3} + \right.$$
$$\left. + \frac{C \exp b\,(b^2 + 2b)}{b + C[\exp b\,(b^2 - 2b + 2) - 2]} \right\}. \tag{21}$$

Since $\alpha_1(T_1)$ is more than 300 times greater than λ_1/b when $T_1(t_1) = 358$ K (Table 1), C is determined from the following quadratic equation (C > 0 corresponds to a physically meaningful solution)

$$C^2 [2(b-1)\exp b + 2]\, b^2 \exp(b) + C\left[2(b-1)\exp b + \right.$$
$$\left. + 2 + \left(\frac{T}{T_*} - 1\right)(b - b^2 \exp b)\right] - 2b\left(\frac{T}{T_*} - 1\right) = 0,$$
$$q = \frac{7{,}75 Q_{28} c\,(G/c)^{0{,}44}}{t \rho_1 c_{p1} \nu} \int_0^a \int_0^b [1 - g\exp(-f\theta)]\, dx dy, \tag{22}$$

$$A = \frac{(T/T_* - 1)(1 - b) + C[2(1 - b)\exp b - 2]}{a^3/3 - a^2 b},\quad \varkappa = \frac{\lambda_1}{\rho_1 c_{p1}},$$
$$\nu = (a + Aa^3/3)\{b + C[(b^2 - 2b + 2)\exp b - 2]\}.$$

Equation (22) is obtained after substituting Eq. (20) into the boundary conditions in Eqs. (14) and (15) and integrating with respect to y from 0 to b and with respect to x from 0 to a, respectively.

With the matching conditions in Eq. (6) [6], Eq. (4) describes the heating of a two-layer plate, while Eq. (2) describes the heating of a concrete chamber wall. They are required for the calculation of the heat losses of walls to external air. Equation (3) is taken from [2]. The Newton conditions in Eqs. (7)-(11) state that heat supplied to the surface of the body by heat transfer is equal to the heat removed from the surface within the body as a result of heat conduction.

Consider the heating in a chamber to which steam is supplied under pressure at one end. At the other end of the chamber is a ventilator with a capacity of 2 m^3/sec. Since the chamber cross section not occupied by the concrete mixture is 1 m^2, the heat-carrier velocity v = 2 m/sec. For the case of induced heat-carrier flow at this velocity, the mean heat-transfer coefficient α_1 was plotted as a function of the air content γ in the saturated steam−air mixture in [2]. Since the air content in saturated steam−air medium at atmospheric pressure corresponds to the temperature of the medium, the dependence $\alpha_1 = f_1(\gamma)$ may also be expressed in the form $\alpha_1 = f_2(T)$ (Table 1).

For practical calculations, the variation of the temperature T in the medium may be expressed as a linear time dependence

$$T = T_{\text{ini}} + \beta t,\ 0 \leq t \leq t_1,\ T = T_{\text{ini}} + \beta t_1,\ t > t_1. \tag{23}$$

Thermal calculation provides the basis for selection of the diameter of the steam-supply tube, the choke diaphragms, the pressure and temperature regulators, the basic elements of the automation system, etc. [2].

The amount of heat consumed in heat treatment in the accelerated hardening of concrete is expressed as the sum of the individual components of the heat consumption [2]

$$Q = (1+\eta)\left(\sum_{i=1}^{5} Q_{i,1} + \sum_{i=1}^{5} Q_{i,2}\right),\ [Q] = \text{kJ}; \tag{24}$$

$$\begin{gathered} Q_{1,1} = \rho_1 V_1 \bar{c}_{p1} (\bar{T}_{1,1} - T_{1\,\text{ini}}) - V_1 q_1,\ Q_{1,2} = \rho_1 \bar{c}_{p1} V_1 (\bar{T}_{1,2} - \\ - \bar{T}_{1,1}) - (q_2 - q_1) V_1,\ q_1 = \text{c}\,Q_{\cdot}(\theta_1),\ q_2 = \text{c}\,Q_{\text{ex}}(\theta), \\ \theta = (\bar{T}_{1,1} + \bar{T}_{1,2} - 2\cdot 273)\, t_2/2 + \theta_1,\ T_{1,1} = (\bar{T}_{1,1} + \\ + T_{1\,\text{ini}})/2,\ \theta_1 = (T_{1,1} - 273)\, t_1; \end{gathered} \tag{25}$$

$$Q_{2,1} = \rho_2 c_{p2} V_2 (T_{1w,1} - T_{1\,\text{ini}}),\ Q_{2,2} = \rho_2 c_{p2} V_2 (T_{1w,2} - T_{1w,1}); \tag{26}$$

$$\begin{gathered} Q_{3,1} = E(T_{1w,1} - T_0),\ Q_{3,2} = E(T(t_1) - T_{1w,1}), \\ E = c_{p4}\rho_4 V_4 + 0{,}6\, c_{p5}\rho_5 V_5; \end{gathered} \tag{27}$$

$$Q_{4,1} = D(\lambda_3 c_{p3} \rho_3 t_1)^{0,5},\ Q_{4,2} = D(\rho_3 \lambda_3 c_{p3} t_3)^{0,5} - Q_{4,1}, \tag{28}$$

$$\begin{gathered} D = 3{,}56\,[T(t_1) - T_0 - 35]\,(S - F_2 + F),\ S = BL,\ F_2 = B_2 L_2; \\ Q_{5,1} = \alpha (S + F)(T_{3H_3} - T_0)\, t_1,\ Q_{5,2} = \alpha (S+F)(T_{3H_3} - T_0)\, t_2, \\ \alpha = 1{,}163\cdot 10^{-3}\,[\xi (T_{3H_3} - T_0)^{0,25} + \varepsilon\sigma (T_{3H_3}^4 - T_0^4)/(T_{3H_3} - T_0)], \end{gathered} \tag{29}$$

$$F = 2LH + 2BH,\ [\alpha] = \text{kJ/(m}^2\cdot\text{sec}\cdot\text{K)},$$

$$\bar{c}_{p1} = (G_{\text{dry}} c_{p\,\text{dry}} + c_{p_{\text{w}}} G_{\text{w}})/(G_{\text{dry}} + G_{\text{w}}),\ G_{\text{dry}} = \text{c} + \text{s} + \text{c}.$$

The mean temperature $\bar{T}_{1,i}$, i = 1, 2, in Eq. (25) is expressed as [2]: $\bar{T}_1 = (T_{1w} + 2T_{1H_1})/3$.

The factor 0.6 in Eq. (27) is introduced to take into account that the temperature of the heating layer is less than the isothermal stage of heating of a concrete mixture. In Eq. (24), η oscillates in the range 0.15-0.3; in Eq. (29), ξ oscillates in the range 1.8-2.2. The coefficient $\eta = 0.15$ corresponds to apparatus with automatic control of the heat-treatment conditions, $\xi = 1.8$ to a horizontal wall surface, and $\xi = 2.2$ to a vertical surface [2].

Since the water–cement number appears in Eq. (3), Q_{ex} is the total thermal effect, in which the contribution of heat absorption on account of the evaporation of water in the volume of the body (i.e., $q_{ex} - q_{va} = Q_{ex}\tilde{C}/t$) is taken into account as well as heat liberation on hydration. This is also confirmed by the good agreement of the theoretical curve in Eq. (3) when $T_1 \leq 363$ K with the experimental data on the heat liberation of Belgorodsk Portland cement as a function of the degree-hours (see Fig. 16 in [2]).

The boundary problem in Eqs. (4)-(10) is solved numerically by means of an implicit monotonic absolutely stable difference scheme on the basis of continuous fitting. Automatic selection of the time step is ensured here on the basis of the specified accuracy, on the one hand, and the convergence of the interactions with respect to T_1, on the other, in view of the nonlinearity of the source in Eq. (1). Supposing that the maximum time step is no more than $0.1t_*$, the time to solve the problem on a BÉSM-6 computer to $t = t_3$ is 0.8 min with a spatial calculation grid $T_2 + T_1(21 + 101)$ and T_3 (121). In addition, the calculation program is tested for an accurate analytical solution [7]. The deviation of the numerical solution from the accurate value in the specified time interval is no more than 0.8%.

Note that the problem is solved in dimensional form. The length of the whole treatment cycle is 12-14 h. Therefore, to minimize the computer time, it is expedient to introduce a characteristic heating time of the concrete $t_* = 3600$ sec (1 hr).

The thermophysical characteristics of the heating chamber are taken from [2], and the geometric dimensions and initial data are specified. Here $T_0 = 278\text{-}258$ K, $T_{i.ini} = 278\text{-}288$ K, i = 1, 2, 3, $\alpha_2 = 1.163$ W/m^2·K, $\alpha_3 = 0.1163$ W/m^2·K, $c_{pdry} = 838$ J/kg·K, $c_{pw} = 4190$ J/kg·K, $\tilde{C} = 200$ kg, $\tilde{S} = 675$ kg, $G_w = 140$ kg, $\tilde{G} = 1500$ kg, $\rho_1 = 2450$ kg/m^3, $\lambda_1 = 1.745$ W/m·K, $c_{p2} = 481.85$ J/kg·K, $\rho_2 = 7800$ kg/m^3, $\lambda_2 = 58.15$ W/m·K, $c_{p3} = 838$ J/kg·K, $\rho_3 = 2300$ kg/m^3, $\lambda_3 = 1.396$ W/m·K, $c_{p4} = c_{p2}$, $\lambda_4 = \lambda_2$, $\rho_4 = \rho_2$, $c_{p5} = 1676$ J/kg·K, $\rho_5 = 200$ kg/m^3, $\beta = 17\text{-}20$ K/hr, $t_1 = 4t_*$, $t_2 = 10t_*$, $t_3 = 14t_*$, $T_s = T_{1.ini}$, $\sigma = 5.7 \cdot 10^{-8}$ W/m^2·K^4, $\varepsilon = 0.9$, $\xi = 2.2$, $\eta = 0.3$, $Q_{28} = 100$, N = 23, $G_w/\tilde{C} = 0.7$, L = 18.7 m, B = 3 m, H = 0.75 m, $l_1 = 0.6$ m, $B_2 = B_1 = 2.4$ m, $H_1 = 0.28$ m, $H_2 = 0.02$ m, $H_3 = 0.3$ m, $H_4 = 0.025$ m, $H_5 = 0.05$ m, $L_2 = L_1 = Nl_1$.

Test Calculation. The amount of heat (steam) consumed in the heat treatment of concrete by a steam–air medium in a cellular chamber is determined from [2] (see pp. 110, 150, for example), using Eqs. (24)-(29), and by numerical solution of the boundary problem in Eqs. (4)-(10). The first column in Table 2 denotes the component of the heat consumption; the second and third columns give the amount of heat obtained from numerical and analytical solution of the problem at the stage of temperature increase of the medium in the chamber; columns four and five give the corresponding results obtained at the stage of isothermal holding. It is evident from Table 2 that there is considerable discrepancy in determining the components of the heat consumption for the heating of concrete with temperature increase of the medium in the chamber and for heating of the metal mold in isothermal holding. In numerical solution of the problem, $T_{1w}(t_1 = 2\,\text{hr}) = 339.67$ K, $T_{1H_1}(t_1 = 2\,\text{h}) = 300.27$ K, whereas the analytical solution from [2] gives 343 and 303 K, respectively. Since the section of temperature increase in the first 2-3 h corresponds to the maximum heat liberation (strongly dependent on the temperature [2]) due to the hydration of cementite [2, 3], these discrepancies make sense. In the second component of the consumption, with isothermal holding, this difference is due to the temperature difference $T_{1w,2} - T_{1w,1}$ in Eq. (26), which is 24.2 and 18.3 in numerical and analytical solution. Note that the contribution of the third component of consumption at the stage of isothermal holding to the total heat balance is slight. In the other components of heat consumption, the error in numerical determination with respect to the analytical result is no more than 10%.

Since a stream tank is considered and the thermal effect of vaporization is $Q_{va} = 2257$ kJ/kg [2], while $Q_{max} = (1 + \eta) \sum_{i=1}^{5} Q_{i,1}/t_1 = 6.5 \cdot 10^5$ kJ/h (in the section of temperature increase of the medium in the chamber), the maximum hourly amount of steam is $m_1 = Q_{max}/Q_{va} = 284$ kg/h. The mean hourly heat consumption in isothermal heating is $\bar{Q} = (1 + \eta) \sum_{i=1}^{5} Q_{i,2}/t_2 = 2.4 \cdot 10^5$ kJ/h ($t_2 = 5$ h). Then the hourly steam consumption in this section is $m_2 = 113.1$ kg/h. Finally, the mean hourly steam consumption in the course of heat treatment ($t_3 = 7$ h) is $m_3 = Q/Q_{va}t_3 = 166$ kg/h.

TABLE 2. Amount of Heat Obtained for Different Components of Heat Consumption $Q_i/4.19$, kJ, in Periods of Temperature Increase and Isothermal Holding of the Medium in the Chamber

i	t			
	1		2	
1	70147	87000	77495	70800
2	89140	95000	41780	31300
3	19400	20600	7537	5630
4	66980	66980	57230	57230
5	34060	34060	81100	81100

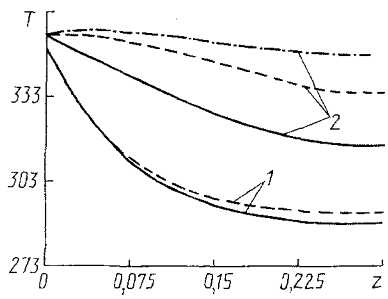

Fig. 1

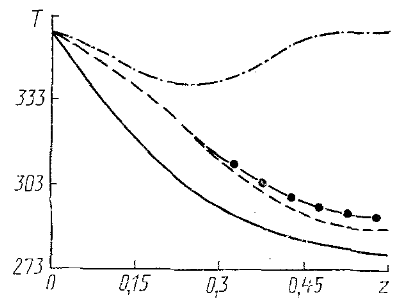

Fig. 2

Fig. 1. Distribution of the temperature, T, K, over the depth z, m, of a concrete chamber wall (continuous curves), mixture on a substrate (dashed curves), and mixture in thermal isolation (dash-dot curve) for times $t = t_1$ (1) and t_3 (2) in case 1.

Fig. 2. Dependence of the temperature T, K, on the depth z, m, of a wall (continuous curve), concrete mixture in ordinary heating conditions (dashed curve), and concrete mixture with substrate heating (dash-dot curve) at $t = t_3$ in case 3; the points correspond to the mean formulation of the problem.

It is impossible to use the analytical expressions from [2] to find the temperature of concrete components with thicknesses $H_1 > 0.3$ m. The surface temperature of the panel in this case at $T = T_{ini} > 273$ K when $t_1 = 2t_*$ has a low or improbable $-$ $T_1(t_1) < 273$ K $-$ value. In addition, these analytical formulas are obtained under the condition that $(\partial T_1/\partial z)|_{z=H_1/2} = 0$, which corresponds to a symmetric temperature distribution over the cross section of the body relative to its axis due to the equal temperatures of the medium on all sides.

ANALYSIS OF THE RESULTS OF NUMERICAL SOLUTION

1. Heating of a Hardening One-Dimensional Plate. First consider the heating of concrete mixture in a chamber in spring–autumn conditions with $T_{ini} = T_{1.ini} = T_{2.ini} = T_{3.ini} = 288$ K, $\beta = 17$ K/h, and $T_0 = 278$ K (case 1). The spatial distribution of the temperature of a concrete wall (continuous curves), a concrete mixture on a steel substrate (dashed curves), and thermally isolated concrete mixture (dash-dot curve) is shown in Fig. 1 for t = 4 (1) and 14 (2) h. It is evident that, with increase in heat-carrier temperature in the chamber, according to Eq. (23), the greatest heating of concrete corresponds to the surface and its immediate vicinity ($z = H_1/4$). When $t = t_3$, the temperature is higher over the whole depth for a concrete

TABLE 3. Consumption of Heat (Steam) for Various Conditions of Heat Treatment of Concrete

Parameters	Conditions of accelerated treatment		
	I	2	3
$Q/4,19$, kJ	$1,0265\cdot 10^6$	$1,054\cdot 10^6$	$1,414\cdot 10^6$
$q/4,19$, (kJ/m³)	$1,4477\cdot 10^5$	$1,613\cdot 10^5$	$1,854\cdot 10^5$
M, kg	1904,5	1956,4	2623,8
m, kg/h	136	139,7	187,4

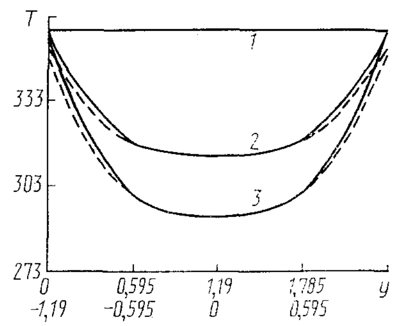

Fig. 3. Distribution of the temperature T, K, over the width y, m, of concrete mixture at $x = L_1/2$ in three cross sections over the thickness: $z_1 = 0$, $z_2 = H_1/2$, $z_3 = H_1$ (continuous curves 1-3) in adiabatic heat-transfer conditions at the lower surface for $t = t_3$; the dashed curves correspond to the mean formulation of the problem.

mixture on a substrate than for a concrete wall (chamber wall). This is due to the heat produced in the exothermal hydration of cement. In adiabatic heat-transfer conditions of concrete mixture, as would be expected, the temperature of the inner surface is 14 K higher than that of concrete mixture on a substrate. Adiabatic conditions are preferable since they correspond to more complete hydration of the cement, which has a significant influence on the mechanical strength of the concrete component [1, 2].

The temperature of the outer surface of the chamber wall at the end of heating $t = t_3$ is $T_{3H_3} = 316$ K, which is 28 K higher than the initial wall temperature. Therefore, to reduce T_{3H_3} and ultimately to reduce the heat losses of the concrete chamber walls, they must be thickened.

Consider the heating of concrete mixture with $T_{ini} = T_{1.ini} = T_{2.ini} = T_{3.ini} = 283$ K, $\beta = 19$ K/h, $T_0 = 278$ K (case 2) and heating of a component in a chamber in winter with $T_{ini} = T_{1.ini} = T_{2.ini} = T_{3.ini} = 278$ K, $\beta = 20$ K/h, $T_0 = 258$ K (case 3). Table 3 gives the total amount of heat – Q in Eq. (24) – as well as the specific heat consumption in heat treatment of the concrete, the heat-carrier consumption in the complete heating cycle, and the hourly consumption of steam in the chamber. Analysis of Table 3 shows that reducing the initial temperature of the concrete mixture by 5 K (case 2), with no change in the other input data, leads to a slight increase in the heat consumed in comparison with case 1, whereas in winter the steam consumption increases by a factor of more than 1.3 and the total amount of heat Q by a factor of 1.4. This is basically due to the heat losses of the concrete chamber wall: the components $Q_{4,i}$ and $Q_{5,i}$, $i = 1, 2$; the latter term may be reduced by increasing (for example, doubling) the thickness of the chamber walls. As a result, the mean steam consumption in the chamber in case 3 when $H_3 = 0.6$ m is reduced to $m = 138$ kg/h.

In case 3, for $H_1 = 0.58$ m and $H_3 = 0.6$ m, the other input data being the same, consider typical heating on a substrate and with heating of the external side of the substrate according to Eq. (23): $T_2 \mid_{z=H_1+H_2} = T_{ini} + \beta t$ $(0 \le t \le t_1)$ and $T_2 \mid_{z=H_1+H_2} = 358$ K $(t_1 < t \le t_3)$. The temperature dependence of a concrete wall, a concrete mixture in steady conditions, and a mixture with substrate heating at $t = t_3$ at any depth is shown in Fig. 2. As would be expected, the temperature of the concrete mixture within the body is considerably higher with substrate heating than in ordinary heating conditions. This is also associated with heat liberation in the hydration of cement, whereas with thermal isolation of the concrete mixture the inner-surface temperature at $t = t_3$ is no more than 292 K.

2. *Heating a Three-Dimensional Concrete Block.* The numerical method proposed in [7] is used to solve the boundary problem in Eqs. (1), (5), (11), and (12). A 21 × 11 × 11 spatial calculation grid is adopted, and the maximum time step is no more than $0.5t_*$. Then the time to calculate the heating of the concrete mixture to $t = t_3$ is 8 min. Consider case 3 of heat transfer in the chamber, when $H_1 = 0.58$ m. The result of solving the problem is shown in Fig. 3 for $t = t_3$. It is evident that, in adiabatic heat-transfer conditions, the temperature of the lower surface at the center $(x = L_1/2)$ reaches 292 K. This indicates that the use of the one-dimensional model of heating is correct in this case.

The results of simplified calculation according to Eqs. (5), (18), (19), and (21) at time $t = t_3$ over the depth of the block is shown by the points in Fig. 2. The temperature of the lower surface of the concrete reaches 292 K here. The dashed curves in Fig. 3 are obtained by Eqs. (20) and (22) when $x = 0$. The deviation from accurate numerical solution of the three-dimensional problem is no more than 9%, which indicates that the simplified formulation is correct.

CONCLUSIONS

Analysis of Eqs. (24)-(29) and the calculation results show that the wall thickness H_3 and the coefficient F_2/S, analogous to the chamber-filling coefficient V_1/V_2, must be increased to reduce the fourth and fifth components of heat consumption in the heat treatment of concrete mixture in a chamber at a polygon [2]. Increasing F_2/S reduces the heat stored by the chamber floor $(S - F_2)$ in Eq. (28). To increase the completeness of cement hydration in concrete components on heating, it is expedient to use a heat-insulating material as the substrate. For thick components $(H_1 > 0.3$ m) in this case, substrate heating is required. The good agreement between the results of solution in the three-dimensional, mean, and one-dimensional formulations confirms the validity of the latter approach and permits corresponding reduction in the computer time required.

NOTATION

x, y, z, spatial coordinates; t, time; T, temperature; c_p, specific heat; ρ, density; G, weight; λ, α, thermal conductivity and heat-transfer coefficient; η, proportion of heat losses that are not taken into account; Q_{ex}, thermal effect of cement hydration; Q_{28} heat liberation in normal 28-day hardening; Q_{va}, heat of vaporization; ω, volume rate of vaporization; $G_w/\tilde{C}$, water-cement number; $\tilde{S}$, $\tilde{G}$, $\tilde{C}$, G_w, weight of sand, gravel, cement, and water, respectively; β, rate of temperature increase of medium in chamber; L, B, H, length, width, and height of heating chamber; l_1, length of a single cell of the metal mold; S, surface area of chamber floor; F_1, surface area of concrete block excluding base; F_2, surface area of steel substrate in contact with block; V, volume; a, b, half the length of the concrete block in the x and y directions; σ, Stefan–Boltzmann constant; ε, surface emissivity of concrete wall; t_1, time of temperature increase of medium in chamber; t_2, time of isothermal holding; t_3, total heating cycle; N, number of steel inserts in metal mold; $Q_{1,i}$, $i = 1, 2$, heat consumed in heating the concrete mixture; $Q_{2,i}$, heat consumed in heating metal mold; $Q_{3,i}$, heat consumed in the heated metal roof; $Q_{4,i}$, heat stored by the walls and part of the chamber floor; $Q_{5,i}$, heat loss of the chamber walls; Q_{max}, maximum heat consumption in conditions of temperature increase; q, specific heat consumption in the heat treatment of concrete; M, consumption of heat carrier in the complete heating cycle; m, hourly consumption of steam in the chamber; γ, steam content per kg of steam–air mixture (γ = 0-1); $\bar{T}$, mean temperature. Indices: 1, 2, 3 (for geometric dimensions and first subscripts of temperature values), concrete mixture, steel substrate, and concrete wall, respectively; w, surface values; H_1, temperature of concrete mixture at contact with substrate; 1, 2 (second indices), period of temperature increase of chamber medium and isothermal holding, respectively (these two subscripts have the same meaning for the thermal losses in Eqs. (24)-(29)); 4, 5, thermodynamic parameters and geometric dimensions of steel roof and heater, respectively; 0, external air; s, temperature under steel floor; dry, dry component of concrete mixture; *, characteristic value; ini, initial value.

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FORMATION OF A THERMAL STRUCTURE IN AN INHOMOGENEOUS METAL CONDUCTOR UNDER A HIGH-DENSITY ELECTRIC CURRENT

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The heating of a metal conductor with a localized inhomogeneous inclusion by a high-density electric current is discussed.

A considerable number of papers on thermal phenomena in nonlinear media have recently been published (e.g., [1-4]). So-called thermal structures, i.e., localized spatially nonuniform steady-state temperature distributions, have been found to form under certain conditions in the regions of space where the processes are fastest. The structures can form when initially the nonlinear medium is spatially uniform and the temperature distribution is nonuniform [1, 2] as well as in the opposite case [3, 4]. The mathematical differences are evidenced by the fact of the dependence of the heat equation explicitly only on the temperature in the first case but also on the spatial variables in the second case. To my knowledge, no one has yet tackled problems of the second type with an analytical approach.

This paper considers the heating of a metal conductor, containing an inhomogeneous inclusion (inhomogeneity), by a high-density electric current. Inhomogeneity is construed as a region of space where the physical characteristics (in the given case, the electrical conductivity of the conductor) differ from those of the ambient medium.

A similar problem was solved in [3, 4] by computer simulation of the medium using a network of nonlinear resistors with randomly distributed inhomogeneities. It was shown that with time these inhomogeneities extend (intergrow) across the current lines. The problem, however, was solved on the assumption of zero thermal conductivity of the medium. The distributions of the electric current and the temperature in a conducting medium with inhomogeneities in the initial stage were considered in [5].

Below we make a qualitative analysis of the dynamics of the intergrowth of a single inhomogeneity in a heat-conducting medium with Joule dissipation of energy, when the electrical conductivity of the inhomogeneity differs little from that of the medium. The mathematical problem is formulated as follows.

An electric current of density **j** flows through an infinite metal medium (conductor) with an inhomogeneity located at

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